



Grade 7/8 Math Circles

March 18-21, 2024

Polynomials - Problem Set

- Determine if each expression is a polynomial. Explain your reasoning.
(a) 5 (b) $\frac{2}{x} - x^2$ (c) $x^4 - 6x + 1$ (d) $3^x - 2x$ (e) $x^{2024} + x^{18} + x^3$
- Simplify each polynomial expression.
(a) $x - 3x^3 + 9x - 4 + 2x^3$ (b) $x^4 - 6x + 1$ (c) $-x - 7x^6 + 5x^4 - 2x^7 - 10x^4 + 5x + x^6$
- Recall the expression from part (e) of Question 1: $x^{2024} + x^{18} + x^3$. What is its dominant term and degree? Is the degree of the overall expression even or odd?
- What is the end behaviour of the polynomial function $y = 2x^{17} - 12x^6 + 9$?
- Explain whether the following statement is true or false:
The polynomial function $y = 5x^3 - 2x^2 + x - 15$ may have 0, 1, 2, or 3 real roots, but since it has a degree of 3, it cannot have more than 3 roots.
- Verify whether the following are roots of the polynomial function $y = 2x^3 + 5x^2 + x - 2$.
(a) $x = 1$ (b) $x = -1$ (c) $x = \frac{1}{2}$ (d) $x = 2$ (e) $x = -2$
- Consider the function $y = x^2 + 2x + 1$. Solve for the root(s) of the function and explain your result. What might the graph look like?
- Are the roots of the function $y = -x^2 - 16$ real or imaginary? Explain how you know without graphing.
- The function $y = -3x^2 + 30x - 75$ has a discriminant of zero and a root at $x = 5$. Explain how we can find the maximum height of the graph of the function without graphing.
- Find the intersection point(s) of the functions $y = 2x^2 - 47x + 9$ and $y = 2x^2 + x + 105$.
- Two sports equipment companies compete to see who can generate the most revenue. Company 1 generates a total revenue R_1 (in thousands of dollars) after each month t based on the function

$$R_1 = 7t^3 + 5t^2 - t - 2$$

while Company 2 generates revenue in thousands of dollars based on the function

$$R_2 = 7t^3 - t^2 + 4t - 1$$



At what time do the two companies generate the exact same revenue?

12. * The Pascal Trading Company sells hockey cards at sports conventions. They price all of their cards at \$5 each. At this price, they can expect to sell 165 cards each day. However, every time they increase the price of their cards by \$1, they sell an average of 15 less cards per day.
- (a) How should the company price each card to generate the maximum possible revenue?
(*Note:* The total revenue is found by multiplying the number of products by the cost of each product.)
 - (b) Each convention is one week long. What is the maximum revenue for the Pascal Trading Company during each convention?